Student Workbook

Unit Review — Conic Sections & Analytic Geometry

Name: ____________________ Date: ____________ Class: ________

UNIT SUMMARY

Consolidate your understanding of ellipses, hyperbolas, parabolas, rotating coordinate axes, and representing conic curves in polar form.

Unit Summary: Conic Sections

$$x^2/a^2 + y^2/b^2 = 1 \mid B^2 - 4AC \mid r = ed/(1 \pm e\cos\theta)$$

10.1 Ellipse — $x^2/a^2+y^2/b^2=1$ (horiz) or $x^2/b^2+y^2/a^2=1$ (vert). $c^2=a^2-b^2$. Vertices $(\pm a,0)$, foci $(\pm c,0)$. $e=c/a$, $0<e<1$.

10.2 Hyperbola — $x^2/a^2-y^2/b^2=1$ (horiz) or $y^2/a^2-x^2/b^2=1$ (vert). $c^2=a^2+b^2$. Asymptotes $y=\pm(b/a)x$. $e=c/a>1$.

10.3 Parabola — $x^2=4py$ (vert) or $y^2=4px$ (horiz). Focus $(0,p)$ or $(p,0)$. Directrix $y=-p$ or $x=-p$. Latus rectum$=|4p|$.

10.4 Rotation — Discriminant B^2-4AC: <0 ellipse, $=0$ parabola, >0 hyperbola. $\cot(2\theta)=(A-C)/B$. Invariants: $A+C$, B^2-4AC.

10.5 Polar Conics — $r=ed/(1\pm e\cos\theta)$ or $r=ed/(1\pm e\sin\theta)$. $e<1$ ellipse, $e=1$ parabola, $e>1$ hyperbola. Focus at pole.

Review Examples

EXAMPLE 1

Identify all features of $x^2/25 + y^2/16 = 1$

- $a^2=25$ so $a=5$; $b^2=16$ so $b=4$
- $c^2=25-16=9$ so $c=3$. Horizontal ellipse
- Vertices $(\pm5,0)$; co-vertices $(0,\pm4)$; foci $(\pm3,0)$; $e=3/5$

Answer: Ellipse: vertices $(\pm5,0)$, foci $(\pm3,0)$, $e = 3/5$

EXAMPLE 2

Find all features of $y^2/9 - x^2/16 = 1$

- Vertical hyperbola; $a^2=9$ so $a=3$; $b^2=16$ so $b=4$
- $c^2=9+16=25$ so $c=5$. Vertices $(0,\pm3)$; foci $(0,\pm5)$
- Asymptotes: $y=\pm(3/4)x$; $e=5/3$

Answer: Hyperbola: vertices $(0,\pm3)$, foci $(0,\pm5)$, asymptotes $y = \pm(3/4)x$

EXAMPLE 3

Find all features of $(x-1)^2 = 12(y+2)$

- Vertical parabola; $4p=12$ so $p=3$
- Vertex $(1,-2)$; focus $(1,1)$; directrix $y=-5$
- Opens upward; latus rectum = 12

Answer: Parabola: vertex $(1,-2)$, focus $(1,1)$, directrix $y = -5$

EXAMPLE 4

Use the discriminant to identify: $3x^2 + 2xy + 3y^2 = 8$

- $A=3$, $B=2$, $C=3$
- $B^2-4AC = 4-36 = -32$, which is less than 0, so Ellipse
- $\cot(2\theta) = (3-3)/2 = 0$ so $\theta = 45°$

Answer: Ellipse; rotate by 45°

EXAMPLE 5

Identify the conic: $r = 8/(2 + 6\cos\theta)$

- Divide by 2: $r = 4/(1 + 3\cos\theta)$
- $e = 3$, which is greater than 1, so Hyperbola. $ed = 4$ so $d = 4/3$
- Vertices: $\theta=0$: $r=4/4=1$. $\theta=\pi$: $r=4/(1-3)=-2$, so $r=2$

Answer: Hyperbola; $e = 3$, vertices at $r = 1$ ($\theta=0$) and $r = 2$ ($\theta=\pi$)

Guided Practice

Work through each problem below. Use the hint if you get stuck — write your final answer on the last line.

1 Write the equation of an ellipse with vertices $(0,\pm10)$ and foci $(0,\pm6)$.

Hint: Vertical ellipse: $a=10$, $c=6$. Find $b^2=a^2-c^2=100-36=64$.

2 Find the asymptotes of $(x+2)^2/9 - (y-1)^2/4 = 1$.

Hint: Center $(-2,1)$; horizontal hyperbola; slopes $\pm b/a = \pm 2/3$.

3 Convert $x^2 + 6x - 8y + 1 = 0$ to standard form.

Hint: Complete the square for x: $(x+3)^2-9=8y-1$ so $(x+3)^2=8(y+1)$.

4 Identify the conic: $4x^2 - 4xy + y^2 + 5x - 10 = 0$.

Hint: $B^2-4AC=16-4(4)(1)=0$ so this is a Parabola.

5 Write the polar equation of an ellipse with $e=1/2$ and directrix $x=-4$.

Hint: Directrix left of pole means use $r=ed/(1-e\cos\theta)$. With $e=1/2$ and $d=4$, $ed=2$.

Quick Review Quiz

Circle the letter of the correct answer for each question.

1 **For $x^2/9 + y^2/25 = 1$, the foci are at:** Multiple Choice

- **A** $(0, \pm 4)$
- **B** $(\pm 4, 0)$
- **C** $(0, \pm\sqrt{34})$
- **D** $(\pm 3, 0)$

2 **The discriminant $B^2-4AC=0$ indicates a:** Multiple Choice

- **A** Circle
- **B** Ellipse
- **C** Parabola
- **D** Hyperbola

3 **For $r = 6/(1 + 2\cos\theta)$, the conic is a:** Multiple Choice

- **A** Ellipse
- **B** Parabola
- **C** Hyperbola
- **D** Circle

4 **For $x^2 = -16y$, the focus is at:** Multiple Choice

- **A** $(0,4)$
- **B** $(0,-4)$
- **C** $(4,0)$
- **D** $(-4,0)$

5 **For $x^2/16 - y^2/9 = 1$, the asymptotes are:** Multiple Choice

- **A** $y=\pm(4/3)x$
- **B** $y=\pm(3/4)x$
- **C** $y=\pm(3/16)x$
- **D** $y=\pm(4/9)x$

Common Mistakes to Avoid

✗ Using $c^2=a^2+b^2$ for ellipses (like hyperbolas).

✓ Ellipse: $c^2=a^2-b^2$ (foci inside, $c<a$). Hyperbola: $c^2=a^2+b^2$ (foci outside, $c>a$).

✗ Confusing the direction of opening for parabolas based on the sign.	✓ $x^2=4py$: $p>0$ opens up, $p<0$ opens down. $y^2=4px$: $p>0$ opens right, $p<0$ opens left.
✗ Forgetting to divide to get standard polar form before reading e.	✓ Always divide numerator and denominator by the constant term first. e is the coefficient of cosθ or sinθ after dividing.
✗ Using $\cot(2\theta)=B/(A-C)$ instead of $(A-C)/B$ for the rotation angle.	✓ The correct formula is $\cot(2\theta)=(A-C)/B$. Numerator is $A-C$, denominator is B.
✗ Thinking the larger denominator in an ellipse is always under x^2.	✓ The larger denominator determines the major axis direction: under x^2 means horizontal, under y^2 means vertical.

Math Tips

★ Conic identification from general form: $B^2-4AC<0$ is ellipse, $=0$ is parabola, >0 is hyperbola. No rotation needed to identify type.

★ Ellipse vs hyperbola: ellipse has + between terms; hyperbola has −. The sign between the fractions is the key.

★ For parabolas: identify which variable is squared (x^2 means vertical, y^2 means horizontal), then find p from 4p=coefficient.

★ Polar conics: divide first, read e, then use e to identify type. cosθ means vertical directrix, sinθ means horizontal directrix.

★ Completing the square: factor out the leading coefficient BEFORE completing the square. Then divide everything by the constant.

★ Asymptote memory for hyperbolas: draw the rectangle 2a by 2b; the diagonals ARE the asymptotes.